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NOTE

Dirac Matrix and Tensor Algebra on a Digital Computer[1]

We report on a collection of subroutines which perform symbolic algebraic manipulations with provisions for the matrix and tensor algebra commonly encountered in the reduction of expressions describing the interactions of elementary particles.

The expressions to be processed are presented in a format closely approximating common notation. In this form, the output is conveniently usable in FORTRAN arithmetic statements for subsequent numerical work. As an example, the expression $((3x - 4y)z^2 - 2)$ might be written ((3.*X − 4.*Y)*Z**2 − 2.). The more complicated expression

$$\{[\delta_{\mu\nu} - p_\mu p_\nu/M^2][\bar{u}_r\gamma_a(\mathbf{r} - \mathbf{k} + m)(1 + i\gamma_5)\gamma_\nu(\mathbf{s} - \mathbf{k} + m)\gamma_a u_s]\},$$

involving vectors p_a, the Dirac matrices γ_a, and $\mathbf{k} \equiv \Sigma_a k_a\gamma_a$, might be coded

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((G(MU,N) − P(MU)*P(N)/M1**2)*($V(R,M)*$G(A)*($/R − $/K + M)*
      (1 + I*$G5)*$G(N)*($/S − $/K + M)*$G(A)*$U(S,M)).
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Scalar, vector, tensor—commuting and noncommuting (denoted above by $)—quantities may be used and are defined by the context in which they appear. Parentheses may be nested arbitrarily deeply.

Subroutines are available to perform the input and output of expressions and the collecting of terms. Multiple substitutions may be made with or without free tensor indices. Tensor-index contraction can be done with the option of distinguishing between covariant and contravariant quantities. Expressions involving products of Dirac matrices may be simplified by anticommuting these matrices and using the Dirac equation. It is possible to evaluate the trace of a product of Dirac matrices, and implied sums of the form $(\gamma_\mu \ldots \gamma_\mu)$ may be done.

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To provide flexibility, the routines are to be called in the order desired by the user in a FORTRAN control program. The operations may be performed repeatedly and in any order. Usual conventions such as the symmetry of $\delta_{\mu\nu}$, the skew symmetry of $\varepsilon_{\alpha\beta\gamma\mu}$, $i^2 = -1$, and the properties of γ_5 are handled automatically. The expressions and equalities to be used are read in as data.

Other programs have been written to perform similar tasks on a computer [1]–[3]. A detailed comparison would require exposition of the internal structure of these programs—a task beyond the scope of a note. Some major differences lie in the ability of ASHMEDAI [4] to process products of large numbers of gamma matrices. In contrast to the program in [3], multiple terms resulting from an operation (e.g., a substitution or solving a trace) upon a single term are produced in time sequence so that traces of products of more than 14 γ's are possible. Also, the facility for anticommuting γ's and applying the Dirac equation allow one to manipulate and simplify expressions before multiplying them together to form traces with the resultant doubling of the number of γ's in an individual product.

The routines are written in machine language for the CDC 3600 for operation under, but with little dependence upon, the PRESTO monitor. A more complete description and decks are available from the author or from the UCDS Computer Center [4].

References

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